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METHOD OF DETERMINING THE PHASE VARIABLES OF THE SOLID PHASE IN DISPERSE FLOWS

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A method was developed for contactless measurement of the kinematic characteristics of the solid phase in disperse flows. The proposed method was substantiated empirically and the error was determined.

Although knowledge of the coordinates and velocity fields of both phases in multiphase systems fully determines the intensity of the target processes in heat and mass transfer, until now they have been little studied due to the lack of experimental methods of investigating them.

This article attempts to develop a method of determining the empirical probability density function for the coordinates and absolute velocity of a test particle in a disperse flow — in particular, in apparatuses with a monodisperse fluidized bed. To do this, we need an empirical method of determining the phase variables of the test particle.

The familiar method in [1, 2] for measuring the coordinates of an isotope-labeled particle has several advantages over other methods [3, 4]: 1) it allows for continuous recording of the position of the particle in the fluidized bed; 2) it permits measurements to be made at any point in the apparatus; 3) the transducers are located outside the apparatus and do not disturb the natural character of flow of the phases; 4) a test particle labeled with the Co^{60} isotope is representative in the sense that, for practical purposes, it is the same in size and weight as the other particles in the monodisperse bed.

However, the method does have several shortcomings, making it impossible to evaluate the particle concentration field in the phase space; instability of the electronic equipment, since the signal is analyzed in analog form; oscillograms based on calibration curves are analyzed manually; it is necessary to differentiate the experimentally obtained coordinates in order to obtain estimates of the absolute velocity of the tagged particle [5].

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In the method we propose for determining the phase coordinates of a test particle tagged with Co^{60}, an attempt was made to eliminate these problems while retaining the main advantages of the method in [1, 2]. We propose to measure not the current in the anode load on the transducer but instead the number of pulses of a certain amplitude from the recording scintillation block over a period of time Δt. We then relate this number to the position and velocity of the particle tagged with Co^{60}. This relation should be in a form that will make it possible to calculate the phase variables on a computer during the experiment.

Physical-Mathematical Foundations of the Measurement Method. Let a particle of granular material tagged with a Co^{60} isotope of activity J_0 be fixed a distance $|r|$ in front of an NaI(Tl) crystal. The random number of current pulses M measured during the time t has the mathematical expectation

$$\langle M \rangle = \frac{\varkappa J_0 d^2 \Delta t}{16} \frac{\exp(-\mu|\mathbf{r}|)}{\mathbf{r}\cdot\mathbf{r}}. \tag{1}$$

If the particle were not stationary, i.e. $r = r(t)$, it is not hard to show that

$$\frac{d\langle M \rangle}{dt} = \frac{\varkappa J_0 d^2 \exp(-\mu)|\mathbf{r}|)}{16\,\mathbf{r}\cdot\mathbf{r}} \left\{1 - t\frac{\mathbf{r}\cdot\dot{\mathbf{r}}}{\mathbf{r}\cdot\mathbf{r}}\ (2+\mu|\mathbf{r}|)\right\}. \tag{2}$$

To find the coordinate $r(t)$ by means of (2) in the absence of information on $\dot{r}(t)$, we will examine the conditions under which we can ignore the second term in the brackets.

According to test data [1, 2], the velocity of particles in a fluidized bed is on the order of 10^{-1} m/sec, while 10^{-2} m < $|r|$ < 10^{-1} m for laboratory apparatuses. It follows from this that the second term in the brackets will be small compared to 1 if $t = 10^{-2} - 10^{-3}$ sec. Henceforth, we everywhere adopted a time interval $\Delta t = 5\cdot 10^{-3}$ sec. Then we use Eq. (2) to obtain a formula for $|r|$:

$$\langle M \rangle \cong \frac{\varkappa J_0 d^2 \Delta t}{16} \frac{\exp(-\mu|\mathbf{r}|)}{\mathbf{r}\cdot\mathbf{r}}. \tag{3}$$

This relation can be simplified by keeping in mind that $\mu \sim 10^0$; $|r| = 10^{-2} - 10^{-1}$ m. Expanding $\exp(-\mu|r|)$ into a series and ignoring the terms beginning with $(\mu|r|)^3$, we obtain the theoretical formula

$$|\mathbf{r}_i| = \frac{-\mu + \sqrt{\mu^2 + 4A_i}}{2A_i}, \quad A_i = \frac{16 M_i}{\varkappa J_0 d^2 \Delta t} - \frac{\mu^2}{2},$$
$$|\mathbf{r}_i|^2 = (a_x^{(i)} - x)^2 + (a_y^{(i)} - y)^2 + (a_z^{(i)} - z)^2. \tag{4}$$

Here, the vector $a_i(a_x^{(i)}, a_y^{(i)}, a_z^{(i)})$ determines the location of the optical center of the i-th transducer-crystal NaI(Tl) in the stationary coordinate system. Since r has three components, then $i \geqslant 3$.

Having measured the number of current pulses M_i in the three transducers, we use Eq. (4) to find x, y, and z. We emphasize that Eq. (4) contains the random number M_i, not its mathematical expectation $\langle M_i \rangle$.

We use Eq. (2) to determine the absolute velocity of the particle $\dot{r} = \dot{r}(\dot{x}, \dot{y}, \dot{z})$, but we take the time t of measurement of the number of current pulses to be larger than in the case of the coordinate determination.

We integrate Eq. (2) from 0 to t; we apply the first law of the mean to the integral containing $\dot{r}(t)$. Then (in the stationary coordinate system)

$$\dot{\mathbf{r}}(\xi)\int_0^t \frac{(\mu|r| + 2)\exp(-\mu|\mathbf{r}|)}{|\mathbf{r}|^4}\, r t dt = -\int_0^t \frac{\exp(-\mu)|\mathbf{r}|)}{|\mathbf{r}|^2}\, dt + \frac{16\langle M \rangle}{\varkappa J_0 d^2},$$

where ξ belongs to the interval (0, t).

If we take $t = n\Delta t$ in (5), we obtain the following algorithm to determine $\dot{r}$. We use (4) to find the coordinates x, y, z for each n successive intervals Δt of seconds, compare the integral sums in accordance with (5), and for $i \geqslant 3$ obtain i linear nonhomogeneous equations relative to estimates of the velocity components $\dot{x}$, $\dot{y}$, $\dot{z}$. We find the absolute velocity from three of them. This algorithm clearly illustrates that the more accurate the estimate of the coordinates, the more accurate the estimate of the velocity components.

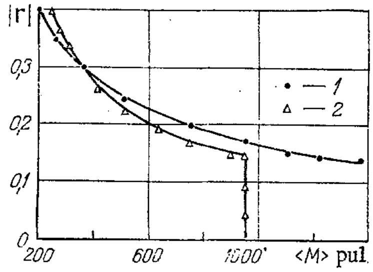

Fig. 1

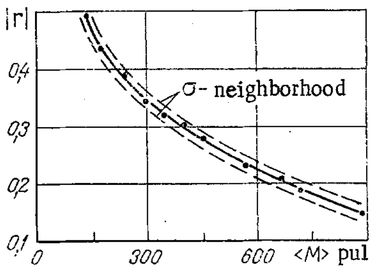

Fig. 2

Fig. 1. Dependence of the number of current pulses in a photocell amplifier on the distance $|r|$ (m), $\Delta t = 5 \cdot 10^{-3}$ sec; $I_0 = 1$ mCi; $d = 0.15$ m; 1) theoretical curve from Eq. (3); 2) empirical curve.

Fig. 2. Calibration relation $\langle M \rangle (|r|)$: $\Delta t = 5 \cdot 10^{-3}$ sec; $I_0 = 1$ mCi; $d = 0.15$ m; σ) estimate of the standard deviation; points denote test results, solid line denotes approximation of the relation $\langle M \rangle (|r|)$.

Conditions of Existence, Uniqueness; Error of the Proposed Method

Analysis of nonlinear system (4), connecting $|r_i|$ $(i \geqslant 3)$ with the three coordinates x, y, z, shows that the condition of the existence and uniqueness of the solution, i.e. of all three values x, y, z, is that the tagged particle should not be located in a plane containing the optical centers of the three chosen crystals NaI(Tl). Let us examine the error of the estimate of the coordinates x, y, z. It naturally depends on the error of the determination of $|r_i|$ (see (4)), while the latter depends on the error of the measurement of M and its closeness to $\langle M_i \rangle$. In fact, theoretical formulas (4) contain not the mathematical expectation of the number of current pulses $\langle M_i \rangle$ in the i-th transducer, but instead the random value M_i, measured over the time $\Delta t = 0.005$ sec.

We will express the increment of $|r_i|$ in (3) through an increment — the pulsation of the measured quantity ΔM_i:

$$|\Delta \mathbf{r}_i| = \left(\frac{\varkappa J_0 d^2 \Delta t}{16}\right)^{-1} \frac{|\mathbf{r}_i|^3 \exp(\mu|\mathbf{r}_i|)}{2+\mu|\mathbf{r}_i|} |\Delta M_i|,$$

where $|\Delta M_i| = |M_i - \langle M_i \rangle|$; $|\Delta r_i| = |r_{i\,\text{true}} - r_i|$. The deviation of the random value M_i from its mean with a probability close to 1 is equal to $|\Delta M_i| \leqslant 3\sqrt{\langle M_i \rangle}$ since M_i is a Poisson process.

We will insert this expression into the previous expression with allowance for $\mu \sim 10^0$. $|r_i| \sim |r_{i\,\text{true}}| \sim 10^{-1}$ m. Then

$$|\Delta \mathbf{r}_i| \cong \frac{6\,|\mathbf{r}_i|^2}{\sqrt{\varkappa J_0 d^2 \Delta t}}. \tag{6}$$

It follows from this that among all of the transducers we need to select that closest to the tagged particle.

Let us examine the effect of the error of calculation of $|\Delta r_i|$ on the error of Δx, Δy, Δz. From system (4) we have

$$(a_x^{(i)} - x)\,\Delta x + (a_y^{(i)} - y)\,\Delta y + (a_z^{(i)} - z)\,\Delta z = |\mathbf{r}_i| \cdot |\Delta \mathbf{r}_i|, \tag{7}$$
$$i = j,\ k,\ l;\ j,\ k,\ l = 1,\ 2,\ 3,\ \ldots$$

Analysis of system (7) with allowance for (6) shows that when the condition of existence and uniqueness of system (4) is satisfied, the error of the coordinate determinations is not strongly dependent on the distances between the transducers and the tagged particle. The magnitude of the error is

$$|\Delta \mathbf{r}| \cong \frac{6}{\sqrt{\varkappa J_0 d^2 \Delta t}},\quad \Delta \mathbf{r}(\Delta x,\ \Delta y,\ \Delta z). \tag{8}$$

We will examine the error of the measurement of the absolute velocity of the tagged particle. Analysis of system (5) shows that the magnitude of the error of $|\Delta \dot{r}|$ is not significantly affected by the determinant of the matrix of the coefficients with the velocity components in the left side of Eq. (5). The greater the absolute value of this determinant, the less the effect of the error of the coordinates of the tagged particle. From this we obtain a method of selecting the optimum trio of transducers out of the total number of transducers. Our experimental unit contained six transducers, and we obtained 20 different third-order determinants. It was necessary to select the ones with the highest modulus in order to find the trio of transducers which would provide the maximum accuracy in determining the absolute velocity.

The following is necessary to reduce the error of the method used here to determine the coordinates and velocity of the tagged particle in a disperse flow: 1) increase the activity J_0 of the isotope in the tagged particle; 2) increase the time of measurement Δt of the coordinate; 3) increase the dimension d of the scintillation crystal; 4) perform the measurements in small apparatuses and locate the NaI(Tl) crystal as close as possible to the wall of the apparatus; 5) from the total number of recording transducers for each coordinate measurement choose the three transducers which ensure the minimum distances to the tagged particle; 6) from the total number of transducers for each velocity measurement choose the three transducers which ensure the maximum modulus of the determinant of system (5).

These requirements, incorporated into an algorithm, can be satisfied even by using a type 15 VSM-5M minicomputer with an external store.

Numerical Experiment. Before developing an experimental unit, we developed an algorithm and a program for a numerical experiment for a BÉSM-4M computer. The essence of the experiment was as follows. We simulated the movement of a tagged particle over a known trajectory $r(t)$ and $\dot{r}(t)$ and generated a Poisson sequence of random numbers with the required mathematical expectation (3). We chose the intensity J_0 = 1 mCi to be as high as possible considering health regulations. The size of the crystal in the "Limon" scintillation block d = 0.15 m, $\Delta t = 5 \cdot 10^{-3}$ sec. The trajectory was in the form of an ellipse with semiaxes D/4 and D = 0.25 m.

The numerical experiment made it possible to significantly improve the accuracy of the method of coordinate and velocity determination if the condition of existence of the solution of system (4) was abandoned (see above); conversely, the closer the tagged particle to the plane of the trio of optical centers of the transducers, the less $|r_i|$ (see (6)) and the more accurate the estimate of r_i. The readings from the trio of transducers are used to calculate only two coordinates, while the third is calculated on the basis of the readings of a transducer not included in the preceding trio but located nearest to the tagged particle.

The absolute error of the coordinate evaluation with such a method of calculation proved to be equal to about 10^{-3} m, while the absolute error of particle velocity components was about 10^{-2} m/sec when the latter were calculated over the time $t = 20\Delta t = 0.1$ sec.

Experimental Verification of the Physical Foundations of the Method

First we checked the theoretical relation (1) and refined the efficiency coefficient of the crystals. The tagged particle was located on the axis of the scintillator at different distances and over $\Delta t = 5 \cdot 10^{-3}$ sec we recorded the number of current pulses in a photocell amplifier.

It follows from Fig. 1 that there is a certain value of $|r|$ equal to 0.1 m such that there is a sharp reduction in $\langle M \rangle$ at distances less than $|r|$; also, the experimental relation $\langle M \rangle$ at $|r| > 0.1$ m differs from the theoretical relation in the character of the change in the function.

Analysis of this result shows that the decisive effect on the change in the operation of the transducers at $|r| \leqslant 0.1$ m is exerted by the fact that the NaI(Tl) scintillator has a fluorescent life of about 10^{-5}-10^{-6} sec. When there is a large flow of γ-quanta, the light flashes occur simultaneously, in packets. Thus, it is impossible to compare the number of light pulses and the number of γ-quanta. This conclusion was confirmed visually by means of an oscillograph. It was found that at $|r| \geqslant 0.17$ m the current pulses do not unite into a single packet, although the method described above for measuring phase variables requires minimum distances between the crystals and the tagged particle.

However, this is not the only correction to the theoretical premises underlying the method. It turns out that the coefficient $\varkappa$ in (1) depends in a complicated manner on the radiant energy, density, and mean atomic number of the scintillator, the geometry of the radiation, the completeness of transmission of light photons to the photocathode of the photocell amplifier, the sensitivity of the discriminator in the electronic circuit, the paths of the γ-quanta in the crystal, the intensity of the Compton scattering, and, thus, objects located around the unit and the voltage supplied to the photomultipliers.

Some of these effects can be accounted for by the relation [6]

$$\langle M \rangle = \frac{J_0 d^2 \Delta t}{16|\mathbf{r}|^2} \exp(-\mu|\mathbf{r}|)\varkappa(|\mathbf{r}|)\exp\left(-\frac{\varkappa(|\mathbf{r}|)J_0 d^2 \tau}{16|\mathbf{r}|^2}\right).$$

But in such a form the function $\varkappa(|r|)$ remains unknown. We thus decided to empirically obtain the relation $\langle M \rangle (|r|)$, approximate it within the limits of the experimental error, and subsequently use it to find the coordinates of the tagged particle. With a fixed value of $|r|$, over the time $\Delta t = 5 \cdot 10^{-3}$ sec we made 1000 measurements of the random number of current pulses M, calculated the mathematical expectation and the dispersion and constructed a histogram. The latter turned out to be close to a Gaussian curve. The calibration curve was obtained in an apparatus with a fluidized bed of alumosilicate catalyst at a gas velocity near to and greater than the critical value. It follows from Fig. 2 that, with a probability of 0.67, the greatest absolute error in the determination of the distance from the tagged particle to the NaI(Tl) crystal within the volume of the apparatus is $\pm 10^{-2}$ m. Thus, this value also establishes the accuracy of the approximation of the relation, which can be used in the algorithm of the program for calculating r(x, y, z) and evaluating the particle concentration field in the volume of the apparatus.

NOTATION

$r(t)$, $\dot{r}(t)$, vector of the coordinate and velocity of the tagged particle; J_0, isotope activity; Δt, measurement time; $\langle M \rangle$, mathematical expectation of the recorded number of γ-quanta; d, $\varkappa$, diameter and efficiency of the NaI(Tl) scintillation crystal; μ, coefficient of linear absorption of the medium; a_i, vector determining the location of the scintillator; τ, fluorescent life of the scintillation crystal.

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